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Characterizing the movement patterns of minibus taxis in Kampala's paratransit system

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ABSTRACT

Urban travelers in Africa depend on minibus taxis for their daily social and business commuting. This paratransit system is loosely regulated, self-organizing, and evolves organically in response to demand. Our study used floating car data to analyze and describe the movement characteristics of nine minibus taxis in Kampala, Uganda. We made three intriguing findings. Firstly, in searching for, picking up and transporting passengers, minibus taxi trajectories followed a heavy-tailed power-law distribution similar to a "Lévy walk". Secondly, their routes' topology and shape gradually changed. Thirdly, the extraordinary winding (expressed in terms of tortuosity) of the paths suggested the extreme determination of the drivers' search for passengers. Our findings could help city planners to build on the self-organizing characteristics of the minibus taxi system, and improve the mobility of travelers, by optimizing routes and the distribution of public amenities.

1. Introduction

In most African countries, minibus taxis are the backbone of public transportation. They transport more than 70% of the total urban travelers and dominate most social and economic aspects of urban mobility (Behrens et al., 2015). They form part of the broader organically evolved paratransit system that operates with little or no regulation in many developing cities of Africa and the Global South (Behrens et al., 2015). Minibus taxi transport is flexible and semi-adaptive, with stops, schedules, fares and routes primarily determined by demand (Lucas et al., 2019). Unlike traditional bus rapid transit (BRT) systems that use buses on fixed routes and schedules developed a long time in advance, minibus taxi drivers in a paratransit system often plan their routes according to the occupancy status of the taxi and anticipated demand (Gauthier and Weinstock, 2010).

The rapid urban population growth (6% per annum) is reshaping urban settlements and changing economic and social population dynamics in Africa (Awumbila, 2017; Mccormick et al., 2016). Coupled with weak and non-transit-oriented city development policies, the population surge in cities will increase the problems of urban sprawl, scattered public amenities and unemployment. The mobility characteristics of urban dwellers will consequently change, triggering a change in minibus taxi movement characteristics in response, and the static minibus taxi route maps proposed by Klopp and Cavoli (Lucas et al., 2019) will no longer be useful. By exploring the evolution of minibus taxi routes in Kampala's paratransit system, our study could pave the way for solutions to the future minibus taxi travel problems.

Minibus taxis rarely get enough passengers to fill up before departure unless they start trips from the major taxi ranks (which are typically few and travelers often shun them). They therefore search for passengers on the way to make the trips profitable. Sometimes, they wait ("hold back") at selected stops in anticipation of passengers turning up, and sometimes, they go off the main route to search for passengers in sparsely distributed places where they anticipate demand for their services. The taxis go up and down the streets in an apparently chaotic fashion, hooting repeatedly, calling out their destinations, randomly inviting pedestrians to board the taxi, and stopping anywhere to tout for potential passengers in total disregard of traffic and municipal laws. We suspected that analysis of these movements of minibus taxis would demonstrate a Lévy walk pattern during the passenger search process (shown in Fig. 1a). A "Lévy walk" (a term we use synonymously with "Lévy flight") is a pattern of movements made by a random walker, where many short movements are randomly interspersed with long ones and occasionally very long ones, as illustrated in Fig. 1b(ii) (Kisaalita and Sentongo-Kibalama, 2007). In Fig. 1a, a minibus taxi moves from origin (O) to destination (D) but in the process makes many detours to

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hunt for passengers in off-route locations L, S, S' and L'.

The Lévy walk theory broadly combines an organisms' need for resources (e.g., food, shelter, or customers) and the need to reduce risks (e.g., from predators or competitors) with the density and renewability of resources to explain the organisms' movement in space. This study focuses on human movement where the minibus taxi driver represents the 'random walker' and the minibus taxi travelers (trips demand) represent the resources being searched for under some predatory risks such as police enforcement, and competition from other minibus taxis on the same route.

In our study we used floating car data (timestamped geo-localization data collected by moving vehicles) to characterize the movement patterns of minibus taxis in Kampala's organically evolved paratransit system. We were interested in discovering whether minibus taxi movement patterns were consistent with Lévy walk behavior; whether the routes the taxis used changed topology or shape over time, in other words evolved; and whether their movements could suggest anything about their level of determination when searching for passengers. This study's primary limitation is the scarcity of floating car data from many minibus taxis in Kampala. Thus, the analysis made in this paper is based on GPS tracking data from nine minibus taxis collected over a continuous period of eight months.

2. The literature and applications to this study

We divided the literature into three categories: the current status of paratransit in African cities, Lévy walk behavior in animal and human movements, and spatial similarity analysis of mobility trajectories.

2.1. Current status of paratransit in African cities

Until recently, the term "paratransit", meaning "beyond standard transit", or "alongside of standard transit", was used (mostly in the US), to refer to supplementary public transport services that do not have fixed routes or timetables but instead respond to travel demand and preferences and are often used by the elderly and the disabled. However, transport researchers have also adopted the term in the context of developing cities of Africa and the Global South to describe the informal transport that is synonymous with public transport in these cities (Behrens et al., 2015). Paratransit in developing African cities is composed of diverse modes, such as minibus taxis (Booysen et al., 2013), tricycle taxis, bicycle taxis (Ndibatya and Booysen, 2020a) and motorcycle taxis (Rhee et al., 2011; Bradbury and Howe, 2002; Ehebrecht et al., 2018). In some African countries, motorcycle taxis dominate the modal share in terms of vehicle composition (e.g., in Lomé, Togo), but minibus taxis dominate the total share of passengers transported per day (Mccormick et al., 2016). In Kampala, for example, the Kampala Capital City Authority (KCCA) estimates that motorcycle taxis comprise 42% of vehicles and carry 9% of people, minibus taxis comprise 21% of vehicles and carry 82% of people. Private cars comprise 37% of vehicles and carry 9% of people (Evans et al., 2018).

There are five main actors involved in minibus taxis system: the owner, the driver, the conductor, the authorities and the users (Booysen et al., 2013; Plano et al., 2020). The owner provides the vehicle, pays for the operating license and is responsible for the maintenance of the vehicle (Mutiso and Behrens, 2011). The driver rents the minibus taxi from the owners at a pre-negotiated daily fee and makes operation-specific decisions such as, when to provide the service, the route for a given trip, and the trip fare depending on the demand and where to stop to pick up passengers (Mutiso and Behrens, 2011). The conductor, if present, is responsible for touting and collecting fares from the passengers (Ndibatya and Booysen, 2020b; Plano et al., 2020). The majority of the paratransit users in Africa's cities are not formally employed and thus tend to have variable and highly irregular commuting schedules and destinations (ITP, 2010). This influences the movement patterns of minibus taxis in the paratransit system.

Several mapping projects have used floating car data to describe the routes taken by the informal paratransit minibus taxis in developing cities, such as Accra (Saddier et al., 2016), Nairobi, Maputo (Lucas et al., 2019), Kampala (Ndibatya et al., 2016), Dar es Salaam and Stellenbosch (Ndibatya et al., 2014). These projects have in some instances produced route maps, such as Digital Matatu for Nairobi (matatu referring to "minibus" in Kenya (Heinze, 2018)) and the Mapas Dos Chapas for Maputo (chapa referring to "minibus" in Mozambique), as well as the standardized data in the general transit feed specification (GTFS) format used by developers to build mobile applications (Lucas et al., 2019). However, the paratransit mapping projects produced static maps, and the researchers ignored the possibility of changes in the routes that would render the maps irrelevant after less than five years. Thus the need to explore the concept of route evolution in the minibus taxi system.

2.2. Lévy walk behavior in animal and human movements

Movement by organisms is a biological process of great significance. In the reviewed literature, researchers have studied biologically motivated movement (searching for habitats, avoiding predators, or foraging) of organisms with cognitive abilities ranging from the relatively simple (e.g. bacteria), to the cognitively complex (e.g. humans), that demonstrates their ability to respond to external stimuli and memorize past movement experiences. The mechanisms by which organisms make movement-related decisions have evolved, as has the biological context that determines the "currency of fitness" or "reward" associated with the movement (such as net food intake, predatory risk, profit, or time savings). To optimize the "currency of fitness", models have been formulated. Of particular interest to us is the "Lévy flight" model developed by Paul Lévy, a French mathematician. He described a particular class of random walks, in which the distance l travelled between events (referred to in this paper as "steps") is drawn from a "heavy-tailed" and scale-invariant probability distribution defined by

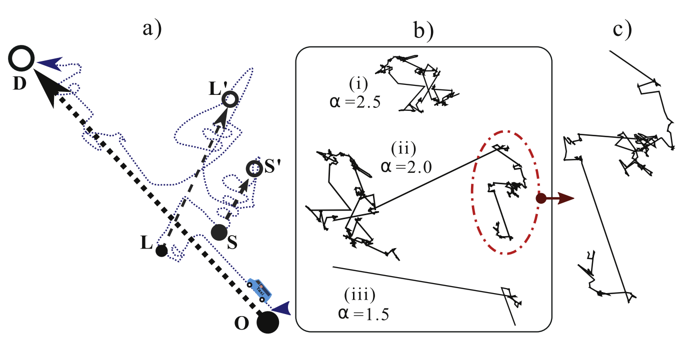

Fig. 1. The concepts of Lévy walk (LW), and minibus taxi movement behavior: a) minibus taxi passenger search behavior from origin (O) to destination (D); b) Lévy walks with different values of Lévy exponent α; c) scale-invariant and fractal properties of a Lévy walk.

Eq. (1) (Viswanathan et al., 1999; Reynolds, 2018).

$$f(l)\tilde{}l^{-\alpha} \quad for \quad l \in [l_{min}, \infty), \tag{1}$$

where l is the step length and α (referred to as the Lévy exponent) is in the range $1 < \alpha \leq 3$ (Viswanathan et al., 2011).

A Lévy walk exhibits three main properties: the probability distribution of step lengths ℓ is heavy-tailed; the turning angles θ between steps are normally distributed, and the step lengths fit strongly into the power-law probability distribution (defined by Eq. (1)). Fig. 1c illustrates the scale-invariant property of a Lévy walk (zooming into a part of a Lévy walk trajectory (in Fig. 1b(ii)) reveals a statistically identical substructure), while the Figures in 1b illustrate the effects of varying values of α on the Lévy walk. Values closer to $\alpha = 1$ lead to ballistic (near-straight) paths (Fig. 1b(iii)), while values closer to $\alpha = 3$ lead to more Brownian behavior (Fig. 1b(i)).

Subsequently, Reynolds (Reynolds, 2015) adapted the Lévy walk theory to explain the movement behavior of cognitively complex organisms in space when searching for patchily and randomly distributed resources. There is empirical evidence of Lévy walks in the movements of foraging birds (albatross (Shlesinger, 2006)), animals (deers (Viswanathan et al., 1999), spider monkeys (Ramos-Fernandez et al., 2003), grey seals (Shlesinger, 2006)), bees (bumblebees (Edwards et al., 2007)) and humans (hunter-gatherers (Raichlen et al., 2014), fishermen (Bertrand et al., 2007), pedestrians (Klopp and Cavoli, 2019)).

Exponents of the Lévy walk theory in the early 2000's sought to explain how organisms optimize their search for sparsely distributed resources (such as food), sometimes under predatory risks. Viswanathan et al. (Viswanathan et al., 1999) combined the Lévy walk model with the optimal foraging approach to formulate and test the Lévy flight hypothesis. This hypothesis states that "since Lévy flights optimize random searches, biological organisms must have therefore evolved to exploit Lévy flights". This paved the way for several predictions of optimal values of α (Lévy exponent in eq. 1) based on the density of the resources a walker is searching for, and how renewable the resources are. Consensus was reached that the optimal value of the exponent α in the Lévy probability distribution, and hence the predicted movement pattern, depends on the "depleting" and "non-depleting" nature of the resource and their density relative to the random walker (Viswanathan et al., 1999; Ferreira et al., 2012). Furthermore, the optimal values of α approach 1 (ballistic movement with little change in direction) for depletable resources. For non-depletable resources, α depends on the target density, for sparsely distributed resources α is closer to 2, and for highly dense resources α is closer to 3 (Brownian motion) (Ferreira et al., 2012).

Lévy walk behavior observed in human movements occur in vast contexts ranging from hunting and foraging among preliterate societies to myriads of contexts among modern and often urban societies. Evidence of Lévy walks in humans predates history as shown in raw material transport distances in the archaeological records (Perreault and Brantingham, 2011). Motivated by the need to search for food, shelter and avoid predators (dangerous animals), preliterate human movements exhibited Lévy walks (Raichlen et al., 2014; Bertrand et al., 2007). Urbanization, industrialization and higher cognitive abilities among humans have diversified the contexts in which Lévy walk behavior can be studied. These contexts are often determined by the travel purpose (such as travel to shop, work, school or leisure), travel mode, and spatial scale (James et al., 2011; Rhee et al., 2008; Brockmann et al., 2006; Cao et al., 2011; Scafetta, 2011). In all these contexts, evidence of Lévy walks has been found. For example, GPS traces from five different outdoor sites (James et al., 2011), circulation of bank notes (Brockmann et al., 2006), city cabs in Beijing (Cao et al., 2011) and long-range human displacements (from 1 to 1000 km) (Scafetta, 2011) exhibited Lévy walks.

However, we did not find any research providing evidence (or absence thereof) of Lévy walk behavior in minibus taxis in a paratransit system. We contend that the minibus taxi movements represent another context in which we can study Lévy walk behavior among humans when searching for non-depletable patchily located and sparsely distributed resources. On the basis of the visual inspection of known minibus taxi trajectories illustrated in Fig. 1a, we hypothesize that while searching for, picking up and dropping off passengers, minibus taxi movement may be consistent with Lévy walk behavior.

2.3. *Spatial similarity analysis of movement trajectories*

The empirical literature on quantifying and analyzing movement trajectory similarity is sparse and scattered across application domains and classes of moving objects. Güting et al. (Güting et al., 2005) identified two classes: objects that maintain a constant shape while moving, such as animals, human beings and vehicles, which they call moving point objects, and those that change their shape, such as a forest fire, which they represent as polygons. This paper is concerned with the former.

The shape of the trajectory is significant. It illustrates how a moving object "winds" its way through a spatial reference system, and it is quantitatively represented in terms of tortuosity, curviness and fractal dimension (Ranacher and Tzavella, 2014). In this study we use only tortuosity (a property of a curve being tortuous or twisted, having many turns, or degree of winding). Researchers use the term "tortuosity" to distinguish between a planned, oriented and effective behavior (low tortuosity), and random search behavior (high tortuosity) (Benhamou, 2004).

We found no studies that quantitatively describe the spatial similarities (and dissimilarities thereof) between minibus taxis' movement trajectories in a paratransit system, hence the need to fill the gap.

The literature we reviewed on the current state of paratransit in Africa, the similarity or dissimilarity between animals and humans that use the Lévy walk search optimization strategy, and the spatial similarity measures of movement trajectories, revealed gaps in paratransit movement-related studies. In addition, the absence of similar studies as applied to minibus taxi movements in a paratransit system, led us to undertake this empirical study. It expands on some of the few existing and limited studies of the operations of minibus taxis in a paratransit system in the Global South (du Preez et al., 2019; Lucas et al., 2019; Ndibatya et al., 2016).

3. Methods

Having acquired and preprocessed the data from the minibus taxis, we used three methods to characterize their movement patterns. We first modelled their trajectories as "walks" composed of sequences of linear steps, defined rules for determining successive steps, and then tested the Lévy walk hypothesis. Secondly, we compared the spatial distances of different minibus taxi trajectories to confirm or refute our route evolution claim. Thirdly, we analyzed the tortuosity (degree of winding) of the trajectories in case it might explain the effort drivers use to search for passengers. We used data collected from a sample of Kampala's taxis. To maximize the accuracy of our results, we assumed that all GPS points were located on the earth's surface, and we computed the distance between them using Vincenty's formula.

3.1. *Data acquisition and pre-processing*

Kampala, Uganda's capital, is home to one and a half million people scattered throughout five administrative divisions: Central, Kawempe, Lubaga, Makindye and Nakawa. Commuters from the latter four and beyond converge mainly in the Central division for work, shopping, leisure and school (Jiang et al., 2009). Minibus taxis, which constitute 82% of the urban public transport, throng the streets of Kampala in a seemingly chaotic pattern, picking up and dropping off passengers at various stops in the city center and the various settlements (Jiang et al., 2009). Many of the stops are informal, i.e. they are not officially designated taxi or bus stops, but develop organically according to the demand in a particular area.

To study the movements of the minibus taxis in time and space, we used standard GPS receivers with a spatial accuracy of three meters and a temporal resolution set to 20 s when the vehicle's ignition is on and 10 min when the ignition is off. The data collected included unique identity, timestamp, longitude, latitude, speed and direction.

We fitted 20 minibus taxis with GPS receivers that transmitted data to our servers for a period of eight months (Jan 2017 to August 2017). For analysis in this paper, we used continuous movement data from nine receivers. Data from other receivers were omitted because of substantial discontinuities due to malfunction, vandalism of the receiver, or frequent mechanical problems with the taxi. Preliminary statistical analysis indicated that the nine minibus taxis under study were active for 155 to 235 days, 12 to 23 h a day, with peak activity occurring between 4:00 and 9:00, 12:00 and 14:00, and 16:00 and 21:00. The mean and standard deviation of days active was 186 and 71 days respectively, while the mean and standard deviation of hours active were 16.3 and 6 h respectively.

To improve performance during analysis, we used MIT's path simplification python library (simplification) to reduce the GPS points while maintaining the integrity of the trajectories. Simplification is a robust high-level implementation of the Ramer-Douglas-Peucker algorithm. Fig. 2a shows a sample minibus taxi trajectory for one day, Fig. 2b the trajectory simplification process as applied to a small section of the original trajectory, Fig. 2c the simplified trajectory, and Fig. 2d the pause-based model that we used to extract Lévy walk steps and turning angles between subsequent steps.

3.1.1. The pause-based model

In a pause-based model (illustrated in Fig. 2d), we define a step as a straight-line movement between two positions P1 and P4 (regarded as pauses) given that the instantaneous velocities at positions P2, P3, and P5 are higher than the threshold velocity. The step length l is the sum of all individual segment distances that make up a step, whereas the turning angle θ is the bearing of the next pause (P6) from the current pause (P4).

From the GPS traces, we extracted the taxis' steps, step lengths, turning angles, and average velocities during the step. To get these data, we re-sampled the trace data every five minutes and re-computed the relative position in space, cumulative distance and average velocity. Using the re-sampled data, we then extracted steps using a pause-based model. Fig. 2e shows the spatial distribution of steps and pauses extracted from a minibus taxi trajectory sample in Fig. 2a.

3.2. The Lévy walk as a descriptor for minibus taxi mobility

To test whether minibus taxis movements exhibit Lévy walks, we divided the minibus taxi trajectories' data into steps and pauses, as described in the pause-based model. We then fitted the step lengths to a power-law distribution defined by a probability density function: $f(l) \sim l^{-\alpha}$ where l is the step length, and α is the Lévy exponent. We then performed a logarithmic transformation on the data and estimated the Lévy exponent α from a power-law fit for each minibus taxi using the power-law python package (Alstott et al., 2014).

3.2.1. Methods to test the minibus taxi Lévy walk behavior

To test the Lévy walk behavior in minibus taxi movements, we performed three different tests on the probability distributions of step lengths and step turning angles. First, we examined the step length distribution's mean spread (standard deviation) and skewness to check if it was heavy-tailed. Second, we examined the distribution of turning angles between steps to establish whether they were normally distributed. Third, we fitted the step lengths' data to a power-law distribution after a logarithmic transformation and estimated the Lévy exponent α to see if it was within the range $1 < \alpha \leq 3$ (the third property of a Lévy walk (Klopp and Cavoli, 2019)). Furthermore, we tested the goodness of the fit using the maximum likelihood estimation method, as suggested by Alstott et al. (Alstott et al., 2014). We did this by comparing the R (log-likelihood) and p (significance) values from the comparison of the best fit of power-law distribution with other distributions, such as exponential distribution.

3.3. Comparing trajectories

We analyzed the similarities and dissimilarities between minibus taxi trajectories by computing their spatial distances from a common fixed position X. We defined a trajectory as the evolution of a minibus taxi's position in space (on the earth's surface) for 24 h; space as the surface of the spheroid earth; position (0.314921, 32.578705) as the latitude and longitude coordinates of the fixed position X, which is a central location at the city square in Kampala; and spatial distance as the unit measure of how far (in space) a one-day trajectory is from a given reference position. Given a trajectory T_1 (illustrated in Fig. 2f) for a day D_1, we

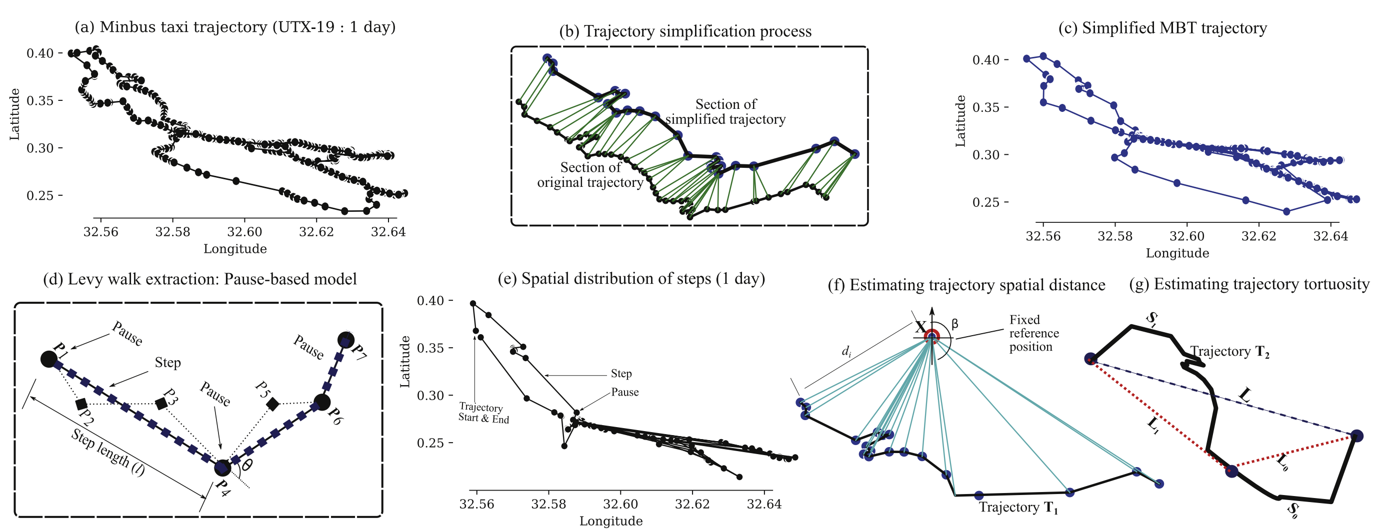

Fig. 2. a) Minibus taxi (MBT) trajectory sample for one day; b) trajectory simplification process; c) simplified trajectory, d) pause-based model to extract steps from a section of the minibus taxi trajectory in a, (e) spatial distribution steps and pauses extracted from the simplified trajectory in (c); f) trajectory spatial distance estimation; and g) trajectory tortuosity estimation.

compute its spatial distance ℓ with respect to an arbitrary fixed reference position $X_{(lon,lat)}$ in space using the equation:

$$\ell = \sum_{i=0}^{n} \sqrt{d_i}\sqrt{\beta_i} \tag{2}$$

where d is the Vincenty distance between the arbitrary fixed GPS position X and the i^{th} GPS point on the trajectory T, and β is the bearing angle between two coordinates $A(A_{lat}, A_{lon})$ and $B(B_{lat}, B_{lon})$ on the earth's surface, given by the equation

$$\beta = atan2(\gamma, \theta) \tag{3}$$

where, $\gamma = \cos(B_{lat}) \sin(|B_{lon} - A_{lon}|)$ and $\theta = \cos(A_{lat}) \sin(B_{lat}) - \sin(A_{lat}) \cos(B_{lat}) \cos(|B_{lon} - A_{lon}|)$.

For each minibus taxi, we normalized the values of all trajectories' spatial distances to fall in the range 0 to 1 in order to simplify the analysis and interpretation of results. Table 2 and Fig. 5a(ii) show the spatial distance distribution for the minibus taxis, sampled per day.

3.4. Trajectory tortuosity

To quantify and analyze the shapes of individual trajectories in order to describe how the minibus taxis wind their way through the spatial reference system, we computed their respective tortuosity values.

We estimated the tortuosity of trajectories as the ratio of a beeline distance between the start and end of the trajectory L to the length of the travelled trajectory S (Grisan et al., 2003) as illustrated in Fig. 2g. For N minibus taxi sub-trajectories in a day's trajectory, the tortuosity τ is computed as:

$$\tau = \frac{N-1}{L} \sum_{i=1}^{N} \left(\frac{L_i}{S_i} - 1 \right) \tag{4}$$

where N is the number of sub-trajectories, L is the beeline distance between the start and end of the day's trajectory, L_i is the beeline distance of the i^{th} sub-trajectory, and S_i is the cumulative length of the travelled sub-trajectory.

4. Results

4.1. Minibus taxi movements and Lévy walk behavior

From the extracted steps, we found that the probability distribution of step lengths is heavy-tailed, as shown in Fig. 3a, with a mean μ and a standard deviation σ of 0.83 and 1.9 km respectively. Its positive skewness of 4.52 shows a significant bulge on the distribution "head" and some rare long walks ("tails") of up to 39 km. This is the first identifying characteristic of Lévy walk behavior (Viswanathan et al., 2011). We also found the second identifying characteristic: turning angles between steps are normally distributed with a mean of 89.4^{o} and a standard deviation of 53.3^{o}, as shown in Fig. 3b.

Micro-level analysis and fitting of individual minibus taxi walks' data to the power-law function revealed a strong power-law behavior for minibus taxis UTX-04, UTX-11, UTX-12, UTX-13 and UTX-17, with an estimated Lévy exponent α in the range $1 < \alpha \leq 3$ (see power-law parameters in Table 1). We further confirmed the power-law behavior in the steps data by comparing the goodness of fit with other distributions and computing the log-likelihood ratio R between the candidate distributions. We also noted the significance value p. Table 1 shows the corresponding values of R and p from the goodness-of-fit comparison

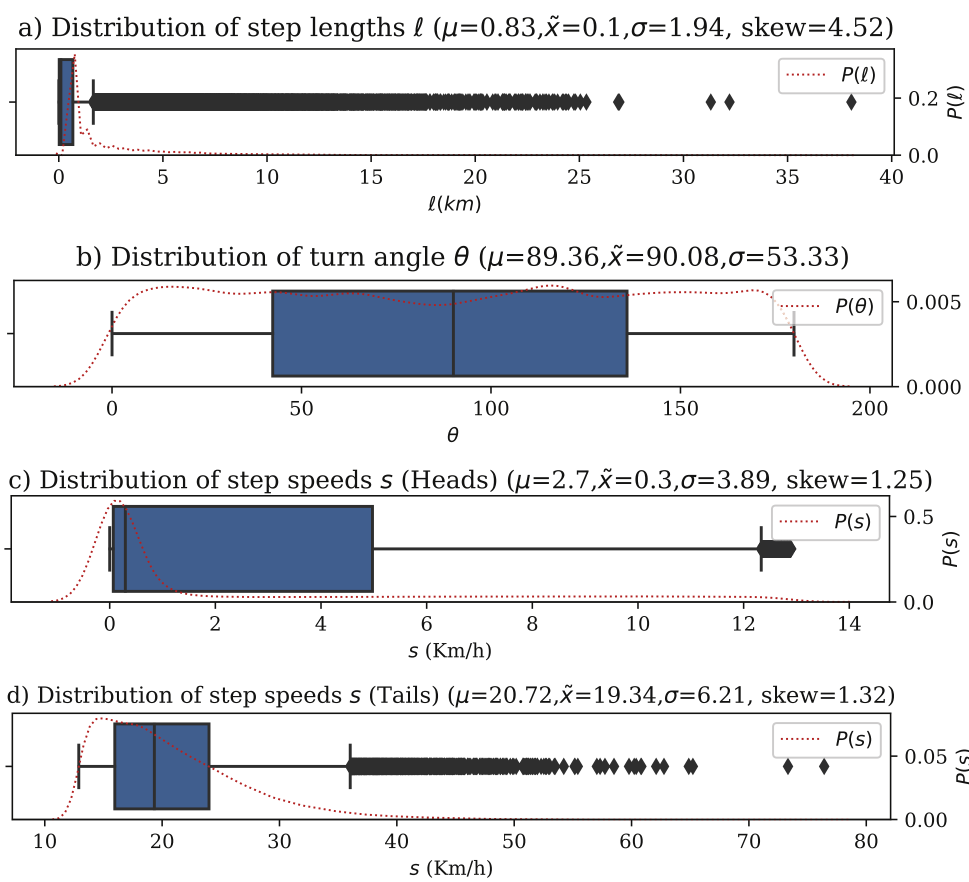

Fig. 3. Summary distributions for all minibus taxis' step lengths, turning angles and steps speeds (speeds were calculated for distances where heads <= 0.676 km, tails > 0.676 km).

Table 1
Step lengths, turning angles, Lévy exponent α and model goodness of fit results.

		Lévy step lengths ℓ (km)				Step turning angles θ^o			Power-law Parameters		Goodness of fit: power-law with exponential		Log-normal	
Taxi ID	#Steps	Max	μ	δ	Skew	μ	δ	Skew	α	σ	R	p	R	p
UTX-04	7887	17.99	1.03	1.79	3.84	89.93	54.19	−0.04	1.98	0.016	9.01	0.070	9.76	0.0
UTX-11	45,924	24.74	0.47	1.61	6.13	91.93	53.83	−0.04	1.53	0.003	101.1	0.120	17.1	0.0
UTX-12	45,947	25.05	0.51	1.84	5.77	89.68	54.38	0.02	1.51	0.003	127.4	0.080	10.5	0.0
UTX-13	18,387	23.81	1.27	2.04	3.30	88.93	57.24	−0.01	1.96	0.009	19.93	0.240	14.9	0.0
UTX-15	7541	18.14	1.21	1.76	3.02	87.71	53.12	−0.02	4.18	0.176	−1.87	0.062	2.27	0.02
UTX-16	13,008	26.92	1.12	1.70	3.34	88.72	47.84	−0.05	4.75	0.159	1.55	0.122	0.65	0.51
UTX-17	2026	20.36	1.52	2.42	2.80	88.86	47.40	−0.04	1.79	0.021	7.50	0.000	6.83	0.0
UTX-18	9515	32.21	1.22	1.94	3.93	87.79	51.81	−0.01	3.86	0.127	1.08	0.279	1.43	0.15
UTX-19	12,780	38.07	1.63	2.92	3.22	85.04	46.19	0.07	7.36	0.521	0.52	0.606	0.18	0.86

between power-law and two other distributions (exponential and log-normal). Figs. 4a(ii), b(ii), and c(ii) compare the goodness of fit of an exponential fit with the power-law fit. The positive R values and p values greater than 0.05 (in Table 1) further confirmed a stronger fit to the power-law than to the exponential and log-normal distributions. Fig. 4 shows that the step lengths data for UTX-04, UTX-13 (4a and 4b) fit the power-law better than the data for UTX-19 do (4c). Fig. 4a(iii), b(iii), and c(iii) further illustrate the strength of power-law fit to the tails (>0.676) of the data. Fig. 4a(iv), b(iv) and c(iv) exhibit a Gaussian mixture of turning angles with multiple Gaussian distributions where each peak represents a major hub visited by the minibus taxi, such as a formal taxi rank, and then makes a sharp turn. We also noted that the multi-Gaussian nature of the turning angle distributions is responsible for the generally high standard deviation of 53.3^o observed in Fig. 3b. (See Fig. 5.)

We concluded that five of the nine minibus taxis under study exhibited Lévy walk behavior. Jiang et al. (Klopp and Cavoli, 2019) say that to identify a Lévy walk pattern, all that is needed is to detect power-law behavior and then estimate the exponent α to see whether it is within the range $1 < \alpha < 3$. For minibus taxis, UTX-04, UTX-11, UTX-12, UTX-13 and UTX-17 the Lévy exponent α for step lengths was in the range $1.51 \leq \alpha \leq 1.98$, and the R values when we compared the power-law function fit with exponential and log-normal model fits were in the range $9.10 \leq R \leq 127.44$. Furthermore, the p values for those five taxis were in the range $0.62 \leq p \leq 0.24$, indicating a strong fit to the power-law, and hence indicating a significant presence of Lévy walk behavior in minibus taxi movement trajectories.

4.2. How similar are the minibus taxis' trajectories?

The spatial distance ℓ_T of a trajectory T, given by eq. 2, is the distance between a fixed position X and the trajectory T, as illustrated in Fig. 2f. We used the observed spatial distances to describe how trajectories from the same minibus taxi differ from each other in space. We took one day as the time interval of each taxi trajectory under discussion. Most of the spatial distances of the minibus taxi trajectories were normally distributed with a mean μ of 0.53 and moderately spread with a standard deviation σ of 0.21. Fig. 5a(i) shows the distribution of normalized spatial distances. Fig. 5a(ii) shows the distributions of distances for each of the nine minibus taxis and Table 2 gives a more detailed breakdown.

We observed a moderate spread in the distribution of spatial distances. This strongly suggests that minibus taxis often divert from the most frequented routes, leading to discovery of new routes, which is also an indicator of growing passenger demand in areas where the new route passes. Practically, if the taxi travelled on the same route all the time, the spatial distances would be less spread. Fig. 5c illustrates how the original route (in 5c(i)) significantly changed over time (to 5c(ii)), then to 5c(iii) in eight months. These changes demonstrate agility and variability,

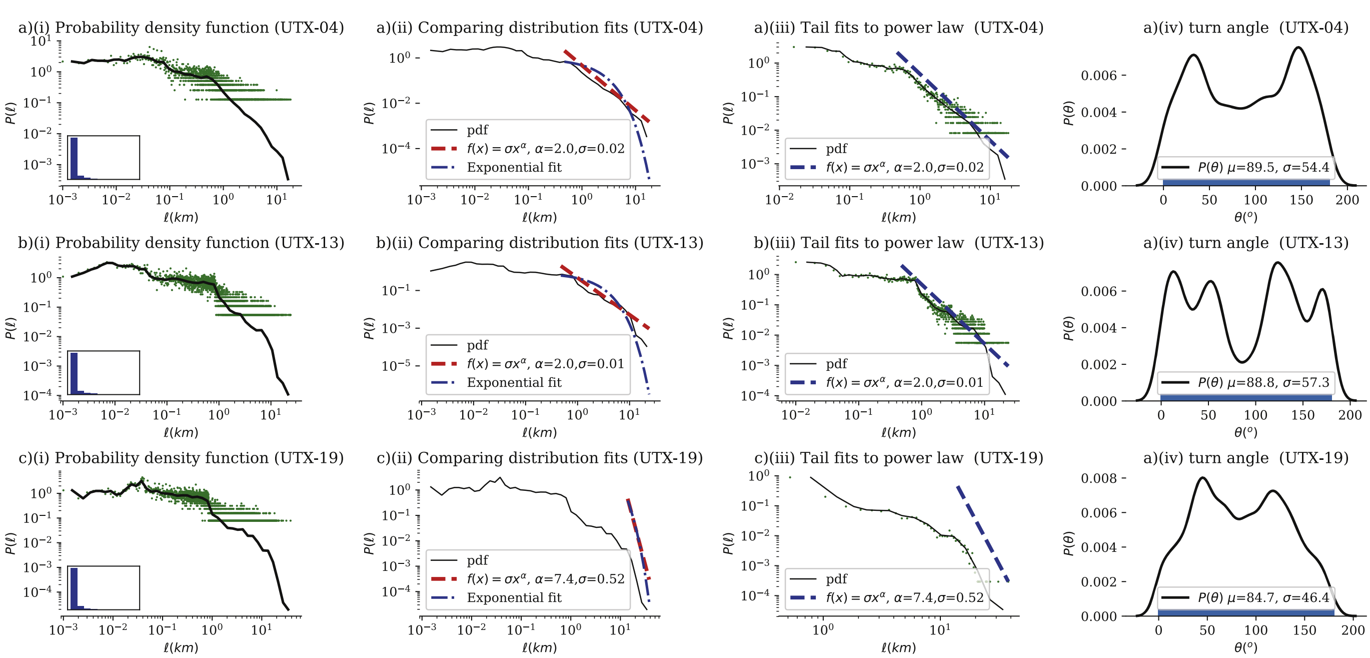

Fig. 4. Power-law analysis of Lévy walk steps.

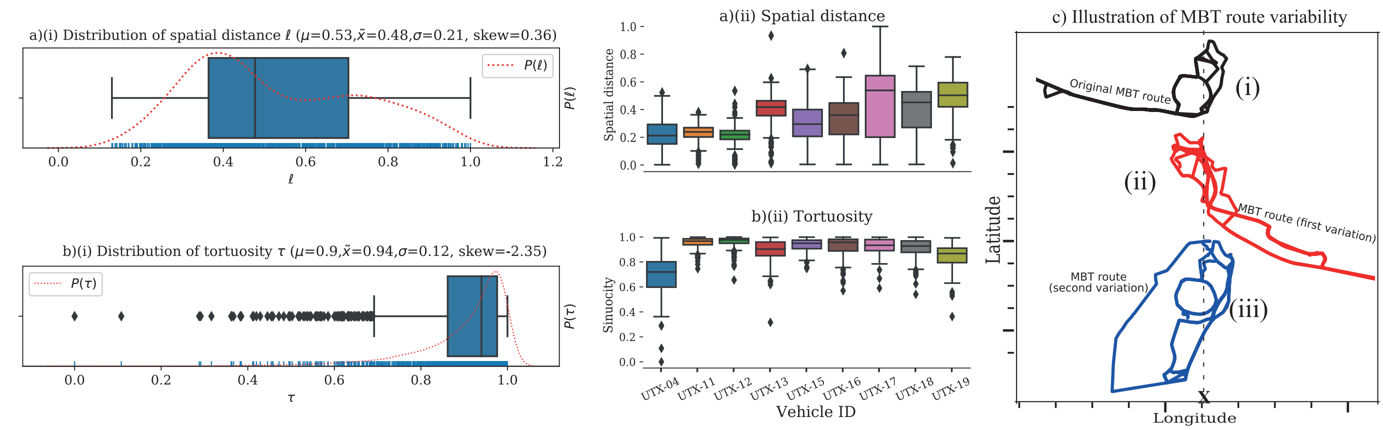

Fig. 5. (a) distribution of normalized spatial distance (b) distribution of normalized tortuosity, (c) illustration of minibus taxi route variability over time, which lends itself to potential evolution.

Table 2
Trajectory characteristics (distance ℓ, tortuosity τ) and Lévy step speeds (heads ≤ 0.67, tails>0.67).

Taxi IDs	#Trjs	Trajectory characteristics				Lévy steps speeds (km/h)						
		SD ℓ		Tortuosity τ		Heads			Tails			
		μ	δ	μ	δ	Max	μ	δ	Min	μ	δ	Max
UTX-04	1983	0.39	0.19	0.68	0.17	76.41	9.53	7.65	3.31	22.73	7.89	52.67
UTX-11	4191	0.40	0.10	0.96	0.04	48.54	0.36	1.03	2.01	14.45	8.09	47.45
UTX-12	4177	0.37	0.11	0.96	0.04	38.43	0.29	0.73	2.00	15.52	9.12	73.33
UTX-13	4806	0.67	0.19	0.90	0.09	53.49	10.51	6.95	2.65	21.30	5.51	57.73
UTX-15	1421	0.50	0.23	0.93	0.06	64.88	10.21	7.34	4.98	21.33	7.00	55.18
UTX-16	3483	0.57	0.24	0.93	0.08	65.23	9.71	6.41	2.05	20.32	5.09	49.30
UTX-17	185	0.44	0.40	0.95	0.06	41.51	10.30	6.35	9.42	20.86	5.58	62.76
UTX-18	1885	0.63	0.28	0.92	0.06	53.42	10.71	6.83	3.04	21.61	6.22	51.07
UTX-19	1728	0.74	0.20	0.85	0.10	51.36	9.71	6.67	2.02	21.11	7.52	55.42

which lends itself to potential evolution.

4.3. *The significance of the observed trajectory tortuosity*

The shape of a trajectory illustrates how a moving object winds or twists its way through a spatial reference system. The similarity of shapes can be expressed qualitatively (topologically), or quantitatively, using parameters such as tortuosity (curviness), and fractal dimension (Ranacher and Tzavella, 2014). We used eq. 4 to estimate the tortuosity τ of the minibus taxi trajectories. The tortuosity values were normalized to fall in the range 0 to 1 and summarized in Table 2. Generally, the tortuosity of minibus taxi trajectories is high as shown by the general distribution in Figs. 5b(i), and at the individual taxi level (5b(ii)). We argue that this tortuosity distribution is suggestive of minibus taxi drivers' extreme determination in searching for passengers to make the trips profitable.

5. Discussion

The results from this study show that minibus taxis movements in Kampala (which represent searching for, acquiring, loading passengers, and transporting them to their destinations) follow a Lévy walk pattern similar to movements observed in a wide range of less cognitively complex species (Shlesinger, 2006; Viswanathan et al., 1999), and recently in humans (Klopp and Cavoli, 2019; Raichlen et al., 2014; Bertrand et al., 2007). The Lévy walk is evident in the many short steps interspersed with rare long steps, and in the Lévy exponent α values (in the range $1 < \alpha \leq 3$) for the greater number of the minibus taxi trajectories (five of the nine sampled taxis). Based on (Viswanathan et al., 1999; Ferreira et al., 2012)'s findings of optimal Lévy walk for undepletable, heterogeneous and patchily located resources, we can claim that two taxis (UTX-04 and UTX-13) had adopted near-optimal search strategies, because they had values of $\alpha \approx 2$ (refer to Table 1). The near-ballistic behavior (α values 1.53 and 1.51) exhibited by taxis UTX-11 and UTX-12 might indicate that the drivers were influenced by previous knowledge of passenger demand (positive memory influence), or it could simply indicate that they often loaded passengers from the taxi ranks. Usually, taxis that load from the taxi ranks take longer to fill up. However, they only load "direct route passengers" going to areas closer to the final destination of the taxi, and they charge a fixed fare equivalent to the maximum amount for the passenger going furthest. We can suggest three possible reasons for the near-Brownian ($\alpha > 3$) movement behavior of minibus taxis UTX-15, UTX-16, UTX-18 and UTX-19. First, the drivers could be new (to the routes, or to taxi driving) and, having not yet figured out a better passenger search strategy, were operating a very inefficient loss-prone strategy. Second, they could be town-service taxis operating in areas with densely distributed informal stops and uniformly distributed short trips demand, leading them to adopt a random search strategy. Third, they might be perennial traffic rule offenders adopting an evasive strategy to avoid encounters with traffic officers on the main routes. This comes at the cost of never being certain of the demand on their "by-pass" routes.

Furthermore, the high tortuosity and moderate spread of spatial distances (in Table 2) suggests that, the drivers search for passengers extremely energetically, some of them aggressively. If the trips are not profitable, one strategy drivers use to improve profitability is to stay at a stop (hold back) until the taxi is full or almost full, and the other is to explore new routes, potentially leading to route evolution. Though we could not verify how profitable the trips were because we lacked data on minibus taxi occupancy, we did a visual inspection of the geospatial layout of minibus taxis routes (using quantum geographical information system (QGIS) software) from individual minibus taxis. From the geospatial layout of the routes (in Fig. 5c), we confirmed the visible change in the shapes of significant routes over several months, and thus concluded that the taxis' routes evolved. Another possible reason for the route evolution is the urban sprawl mentioned earlier. With the proliferation of informal settlements, and poor planning for the locations of amenities like schools, hospitals and shopping centers, the passenger demand is sparsely distributed among sparsely populated patches around the city, hence the unstable transport supply characteristics visible in route changes.

6. Conclusion and recommendations

We have demonstrated, using a sample of nine minibus taxis in Kampala, that minibus taxi movements in a quasi-demand-responsive paratransit system exhibit features statistically similar to those of a Lévy walk. We argue that even the minibus taxis that showed features outside the Lévy walk parameters will eventually subconsciously adopt the Lévy walk strategy. This, we suspect, is because of memory influence (ability to learn, memorize and respond to passenger demand), and the need to optimize profits. Our research further showed that significant minibus taxi routes evolved (changed topology and shape) with time, which is suggestive of the dynamic demand patterns, and the demand-responsive nature of the minibus taxi paratransit system. Finally, we found that minibus taxi routes were extremely tortuous, indicating a determined, energetic, and even aggressive search for passengers. The stress of this kind of search could aggravate driver tempers, encouraging dangerous driving behavior and leading to road accidents. Our research, therefore, could be useful for urban planners interested in transit-oriented city planning, to optimize land use and distribution of the public amenities which are the primary origins and destinations of passenger trips. Furthermore, a transit-oriented city and passenger demand plan could reduce the tortuosity of routes, substantially optimize the Lévy exponent, thereby reducing aggression during passenger search and producing less frustrated, happier drivers and consequently a safer paratransit system. This research could also be useful for modeling purposes, especially agent-based modeling of minibus taxis in paratransit systems.

We were unable to conclusively verify the effectiveness of the Lévy walk strategy in minibus taxis because we lacked data on minibus taxi occupancy. However, based on the Lévy exponent α, we suspect that the minibus taxi search strategies revealed by our data were inefficient. This is because only two minibus taxis had a close to optimal Lévy exponent (UTX-04, UTX13, with values of α=1.98 and α=1.96). We can further conclude that the other seven minibus taxis passenger search strategies were not adquate, pointing to a rather inefficient paratransit system.

For further research, we recommend topics involving the spatial-temporal distribution of passenger trips (trip demand), the influence of predators (police and competing minibusses) on the values of α, and their subsequent relationship with occupancy and profitability as indicators of passenger search strategy effectiveness and optimality. We also foresee further analysis being done to fully characterize and demonstrate route evolution.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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